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MATHEMATICS 4016
PAPER 2 SOLUTIONS

1 **(a)** **(i)** Time spent $= \dfrac{3}{2+3+7} \times 1\dfrac{1}{2} = \dfrac{3}{8}$ h

(ii) $\dfrac{7}{2+3+7} \times 100\% = 58\dfrac{1}{3}\%$

(b) **(i)** Speed $= \dfrac{3000 \div 1000}{9\frac{1}{2} \div 60} = 18\dfrac{18}{19}$ km/h

(ii) $90\% \times 9\dfrac{1}{2} = 8\dfrac{11}{20}$ min $= 8$ min 33 s

(iii) $\dfrac{100}{95} \times 9\dfrac{1}{2} = 10$ min 0 s

2 **(a)** **(i)** $25 - p^2 = 5^2 - p^2 = (5+p)(5-p)$

(ii) $\dfrac{25-p^2}{15+3p} = \dfrac{(5+p)(5-p)}{3(5+p)} = \dfrac{5-p}{3}$

(b) $\dfrac{3}{(x+2)^2} - \dfrac{4}{x+2} = \dfrac{3-4(x+2)}{(x+2)^2} = \dfrac{3-4x-8}{(x+2)^2}$

$= \dfrac{-5-4x}{(x+2)^2}$

(c) **(i)** $v^2 = u^2 - 2gh$

$v = \pm\sqrt{u^2 - 2gh} = \pm\sqrt{30^2 - 2(9.8)(24)}$

$= \pm 20.7$

(ii) $v^2 = u^2 - 2gh$

$u^2 = v^2 + 2gh$

$u = \pm\sqrt{v^2 + 2gh}$

3 **(a)** $BC = 12\sin 20° = 4.1042$

$BD = BC + CD = 4.1042 + 6 = 10.1042 \approx 10.1$ m

(b) **(i)** $GN^2 = AG^2 - AN^2 = 12^2 - 5^2 = 119$

$GN = \pm\sqrt{119}$ (reject negative value)

$GN = \sqrt{119} = 10.9$ m

(ii) $G\hat{A}N = \cos^{-1}\left(\dfrac{5}{12}\right) = 65.376°$

Angle through which the jib has rotated

$= 65.376° - 20° = 45.376° \approx 45.4°$

4 **(a)** **(i)** $B\hat{E}C = B\hat{A}C = 35°$

(angles in the same segment)

(ii) $B\hat{A}E = B\hat{A}C = 35°$

$E\hat{B}C = E\hat{C}B - B\hat{E}C = 90° - 33° = 57°$

(iii) $C\hat{D}E = 180° - E\hat{B}C = 180° - 57° = 123°$

(opposite angles of a cyclic quadrilateral)

(b) In triangles FAE and FDC,

$A\hat{F}E = D\hat{F}C$ (common angle)

$F\hat{A}E = 90° - 33° = 57° = 180° - 123° = F\hat{D}C$

$A\hat{E}F = D\hat{C}F$ (angle sum of a triangle)

By the AAA property, triangles FAE and FDC are similar.

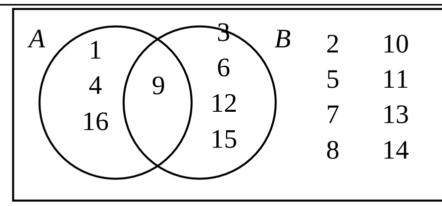
(c) Since triangles FAE and FDC are similar,

$\dfrac{DF}{CF} = \dfrac{AF}{EF} = \dfrac{AC+CF}{ED+DF} = \dfrac{4}{3} = \dfrac{AC+3}{8+4}$

$4(8+4) = 3(AC+3)$

$AC \dfrac{4\times 12}{3} - 3 = 16 - 3 = 13$ cm

5 **(a)** **(i)**



(ii) $A' \cap B = \{3, 6, 12, 15\}$

(iii) $n(A \cup B) = 8$

(b) **(i)** $\mathbf{D} = \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix}$

(ii) $\mathbf{E} = 5\mathbf{C} + \mathbf{D} = 5\begin{pmatrix} 10 & 30 \\ 20 & 10 \end{pmatrix} + \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix}$

$= \begin{pmatrix} 50 & 150 \\ 100 & 50 \end{pmatrix} + \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix} = \begin{pmatrix} 75 & 165 \\ 110 & 80 \end{pmatrix}$

(iii) The elements of **E** represent the total number of adults and children carried by bus from Monday to Saturday.

(iv) **(a)** $\mathbf{F} = \mathbf{C}\begin{pmatrix} 25 \\ 15 \end{pmatrix} = \begin{pmatrix} 10 & 30 \\ 20 & 10 \end{pmatrix}\begin{pmatrix} 25 \\ 15 \end{pmatrix} = \begin{pmatrix} 700 \\ 650 \end{pmatrix}$

(b) The elements of **F** represent the total fare received, in cents, from both adults and children on a weekday morning and a weekday afternoon.

(c) $\mathbf{G} = \dfrac{1}{100}\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{F} = \dfrac{1}{100}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 700 \\ 650 \end{pmatrix}$

$= \dfrac{1}{100}\begin{pmatrix} 1350 \end{pmatrix} = \begin{pmatrix} 13.50 \end{pmatrix}$

(d) **G** represents the total fare received, in dollars, from both adults and children on a weekday.

6 **(a)** $y=\frac{1}{10}\left(60-x^2-\frac{80}{x}\right)=\frac{1}{10}\left(60-6^2-\frac{80}{6}\right)$

$=1\frac{1}{15}\approx 1.07$ (2 dp)

(b) *Refer to graph*

(c) $y=\frac{1}{10}\left(60-x^2-\frac{80}{x}\right)=2$

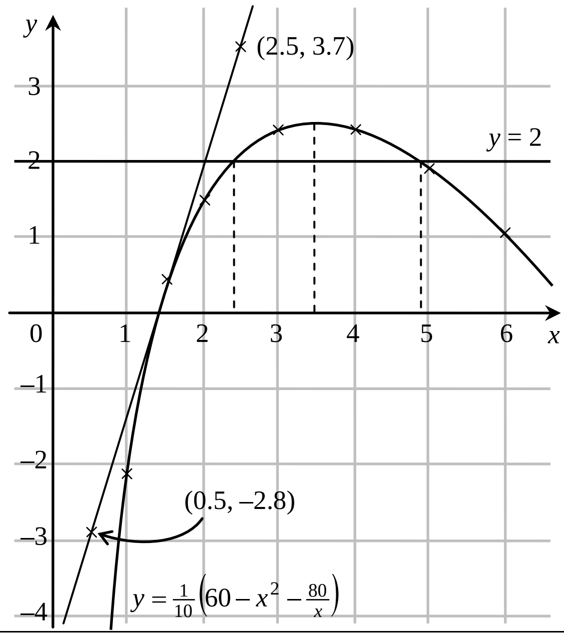
$x=2.3$ or $x=4.85$

(d) *Refer to graph.*

Gradient $=\frac{3.7-(-2.8)}{2.5-0.5}=\frac{6.5}{2}=3.25$

(e) Largest value of $y=2.5$

The value of x for which this occurs is $x=3.4$



7 **(a)** Let M be the midpoint of AB.

$A\hat{O}B=\frac{360°}{5}=72°$

$A\hat{O}M=\frac{72°}{2}=36°$

$OM=\frac{2}{\tan 36°}=2.753\approx 2.75$ cm

(b) Radius of outer circle $=OM+2$

$=2.753+2=4.753\approx 4.75$ cm

(c) Total length of wire needed to make the ornament

$=2\pi\times 4.753+5\times\frac{1}{2}\times 2\pi\times 2+5\times 4=81.3$ cm

(d) Area enclosed by the wire pentagon

$=5\times\frac{1}{2}\times AB\times OM$

$=5\times\frac{1}{2}\times 4\times 2.753=27.53\approx 27.5$ cm

(e) Area of the region shaded

$=\frac{\pi\times 4.753^2-5\times\frac{1}{2}\times\pi\times 2^2-27.53}{5}=2.404$

$\approx 2.40\text{ cm}^2$

8 **(a)** **(i)** $B\hat{A}C=100°-62°=38°$

$BC^2=AB^2+AC^2-2(AB)(BC)(\cos B\hat{A}C)$

$=550^2+645^2-2(550)(645)(\cos 38°)$

$=159431.4$

$BC=\sqrt{159431.4}=399.29\approx 400$ m

(ii) $\frac{\sin A\hat{C}B}{AB}=\frac{\sin B\hat{A}C}{BC}$

$\frac{\sin A\hat{C}B}{550}=\frac{\sin 38°}{399.29}$

$\sin A\hat{C}B=\frac{550\sin 38°}{399.29}=0.84804$

$A\hat{C}B=\sin^{-1}0.84804=57.999\approx 58.0°$

(iii) $A\hat{B}C=180°-38°-58.0°=84.0°$

Bearing of C from B

$=360°-(180°-62°)-84.0°=158.0°$

(b) **(i)** $AD=645\tan 7°=79.196\approx 79.2$ m

(ii) Let x be the shortest distance from A to BC.

$\frac{1}{2}\times BC\times x=\frac{1}{2}\times AB\times AC\times\sin 38°$

$\frac{1}{2}\times 399.29\times x=\frac{1}{2}\times 550\times 645\times\sin 38°$

$x=\frac{550\times 645\times\sin 38°}{399.29}=546.99$ m

Greatest possible angle of elevation of D from a point on BC

$=\tan^{-1}\left(\frac{79.196}{546.99}\right)=8.238\approx 8.2°$

9 **(a)** $ST=AB-AT-SB=AB-TP-SQ$

$=17-x-7=(10-x)$ cm

(b) **(i)** $PQ=(7+x)$ cm

(ii) $QR=(7-x)$ cm

(c) $PQ^2=PR^2+QR^2=ST^2+QR^2$

$(x+7)^2=(10-x)^2+(x-7)^2$

$x^2+14x+49=100-20x+x^2+x^2-14x+49$

$x^2-48-100=0$

(d) $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-48)\pm\sqrt{(-48)^2-4(1)(100)}}{2(1)}$

$=45.82$ or 2.18

(e) We reject $x=45.82>17$, so $x=2.18$.

Diameter of the smaller cylinder

$=2(2.1826)=4.3652$ cm $=43.652$ mm ≈ 44 mm

10 **(a)** **(i)**

Score	4	5	6	7	8	10
Frequency	2	5	11	8	3	1

(ii) **(a)** Mean score $= \dfrac{\Sigma fx}{\Sigma f} =$

$\dfrac{4\times 2+5\times 5+6\times 11+7\times 8+8\times 3+10\times 1}{30}$

$= \dfrac{189}{30} = 6.3$

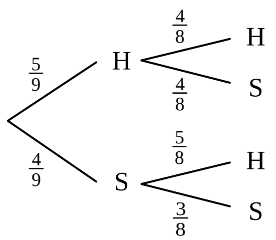
(b) $\Sigma x^2 = 2\times 4^2 + 5\times 5^2 + 11\times 6^2 + 8\times 7^2$

$+3\times 8^2 + 1\times 10^2 = 1237$

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \left(\dfrac{\Sigma fx}{\Sigma f}\right)^2}$

$= \sqrt{\dfrac{1237}{30} - \left(\dfrac{189}{30}\right)^2} = 1.2423 \approx 1.24$

(b) **(i)** Let H and S represent a chocolate with a hard and soft centre respectively.



(ii) **(a)** $P\left(\begin{array}{l}\text{Ann and Ben both choose a}\\ \text{chocolate with a hard centre}\end{array}\right)$

$= \dfrac{5}{9}\times\dfrac{4}{8} = \dfrac{5}{18}$

(b) $P\left(\begin{array}{l}\text{Ben chooses a chocolate}\\ \text{with a soft centre}\end{array}\right)$

$= \dfrac{5}{9}\times\dfrac{4}{8}+\dfrac{4}{9}\times\dfrac{3}{8} = \dfrac{4}{9}$

(c) $P\left(\begin{array}{l}\text{One chooses a chocolate with a}\\ \text{hard centre and the other chooses}\\ \text{a chocolate with a soft cenre}\end{array}\right)$

$= \dfrac{5}{9}\times\dfrac{4}{8}+\dfrac{4}{9}\times\dfrac{5}{8} = \dfrac{5}{9}$

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