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MATHEMATICS 4016
PAPER 1 SOLUTIONS

1 **(a)** $20xy - 5y = 5y(4x - 1)$

(b) $4x - 4(x+3) = 4x - 4x - 12 = -12$

2 $48 \times \frac{100+135}{100} = 112.8$

The selling price of the watch is \$112.80

3 **(a)** Blue

(b) $\frac{5}{9+5+1+6+3} \times 360° = 75°$

4 **(a)** $\frac{17}{24} \times 100\% = 70\frac{5}{6}\%$

(b) Fraction of candidates who were not awarded an A or B grade $= 1 - \frac{1}{3} - \frac{1}{4} = \frac{5}{12}$

5 **(a)** $2^p \times 5 = 40$

$2^p = \frac{40}{5} = 8 = 2^3$

$p = 3$

(b) $1 \div x^{-4} = x^4$

6 **(a)** $x = 180 - 130 = 50$

(b) Sum of interior angles is $(5-2) \times 180° = 540°$

$y = 540 - 50 - 130 - 80 - 130 = 150$

7 $-2 < 2x - 5 < 7$

$-2 < 2x - 5$	$2x - 5 < 7$
$-2 + 5 < 2x$	$2x < 5 + 7$
$3 < 2x$	$2x < 12$
$1.5 < x$	$x < 6$

$1.5 < x < 6$

8 $5000 \times 1.048^6 = 6624.27$

$6624.27 - 5000 = 1624.47$

Total interest earned is \$1624.27

9 **(a)** Area of triangle ABC

$= \frac{1}{2} \times 7.43^2 \times \sin 38° = 17.0 \text{ cm}^2$

(b) Volume of prism

$= 16.99 \times 20 = 339.88 \text{ cm}^2 = 340 \text{ cm}^2$

10 **(a)** Second term = 38 – 7 = 31

Third term = 31 – 7 = 24

(b) $45 - 7n$

11 For the small tin, cost per unit mass is

$= \frac{1.04}{415} = \$0.00251/\text{g}$

For the large tin, cost per unit mass is

$= \frac{1.98}{815} = \$0.00243/\text{g}$

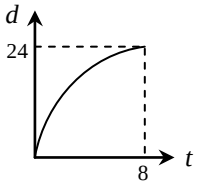
The **larger** tin gives the better value. It has a lower cost per unit mass, where 0.00243 < 0.00251.

12 **(a)** $a = \frac{24}{40} = \frac{3}{5} \text{ m/s}^2$

(b) Total distance travelled $= \frac{1}{2} \times 60 \times 24 = 720 \text{ m}$

13 **(a)** $t = \left(\frac{12}{24}\right)^2 \times 8 = 2$

(b)



14 Surface area

$= 2\pi r^2 + \pi rl = 2\pi(2.8)^2 + \pi(2.8)(7.2) = 113 \text{ cm}^2$

15 **(a)** **(i)** $\sqrt[3]{640} : \sqrt[3]{1250} = 4 \times \sqrt[3]{10} : 5 \times \sqrt[3]{10} = 4:5$

(ii) $4^2 : 5^2 = 16:25$

(b) $25 \times \frac{640}{1250} = 12\frac{4}{5} \text{ kg}$

16 **(a)** **(i)** $AC = 2 \times 8 \sin\left(\frac{60°}{2}\right) = 16 \sin 30° = 8 \text{ cm}$

(ii) $A\hat{B}C = 360° - 60° - 2 \times 130° = 40°$

(b) **(i)** $P\hat{O}T = 2 \times T\hat{P}Q = 2 \times 32° = 64°$

(ii) $O\hat{T}P = 90° - 64° = 26°$

17 **(a)** **(i)** $2(3)^2 + k(3) - 15 = 0$

$3k = 15 - 18 = -3$

$k = -1$

(ii) $2x^2 - x - 15 = 0$

$(x-3)(2x+5) = 0$

$x = 3 \text{ or } x = -\frac{5}{2}$

The other solution is $x = -\frac{5}{2}$.

(b) $6p^2 - 3pq - 10ap + 5aq$

$= 3p(2p - q) - 5a(2p - q) = (3p - 5a)(2p - q)$

18 **(a)** $150 = 2 \times 3 \times 5^2$

(b) $48 = 2^4 \times 3$

HCF of 150 and 48 is $2 \times 3 = 6$

(c) LCM of 150 and 48 is $2^4 \times 3 \times 5^2 = 1200$

Least number of packs of chocolate bars John bought is $\frac{1200}{150} = 8$

19 **(a)** **(i)** $\frac{494.6}{56.33 \times 98.12} = 0.08948635556$

(ii) 0.1

(b) $280 \tan 30° = 161.7 \approx 162 \text{ m}$

20 **(a)** Median mark = 54

(b) Interquartile range = 65 – 44 = 21

(c) Number of students = 800 – 720 = 80

21 **(a)** $1.32\times10^9 - 832\times10^6 = 4.88\times10^8$

(b) $\dfrac{832\times10^6}{26.6\times10^6} = 31.3$

(c) $n = \dfrac{1.32\times10^9}{4.48\times10^6} = 294\dfrac{9}{14}$

Ratio is $1 : 294\dfrac{9}{14}$

22 **(a)** $s = r\theta = 10(2\pi - 2.3) = 39.8\text{ m}$

(b) $x = \dfrac{\pi}{6}$ or $x = \dfrac{5\pi}{6}$

23 **(a)** $x + 2y + 80 = 180$

$\Rightarrow x + 2y = 100$ ——— (1)

$x + 3y + 100 + 129 = 360$

$\Rightarrow x + 3y = 131$ ——— (2)

(b) $(2) - (1)$: $y = 31$

$x = 100 - 2y = 100 - 2\times31 = 38$

24 **(a)** In triangles *ABX* and *DCX*,

$AX = DX$ (given)

$BX = CX$ (given)

$A\hat{X}B = D\hat{X}C$ (vertically opposite angles)

By the SAS property, triangles *ABX* and *DCX* are congruent.

(b) Triangles ABD and DCA
OR Triangles ABC and DCB

(c) Triangles AXD and CXB

25 **(a)** Gradient $= \dfrac{3-1}{6-0} = \dfrac{2}{6} = \dfrac{1}{3}$

(b) $y = \dfrac{x}{3} + 1$

(c) $\dfrac{1}{2}\times(1-(-2))\times6 = 9\text{ units}^2$

(d) (6, 0), (6, 6), (–6, –4) **(any 2)**

26 **(a)** **(i)** $\overrightarrow{BM} = \dfrac{1}{3}\overrightarrow{BA} = \dfrac{1}{3}(6\mathbf{a} - 6\mathbf{b}) = 2\mathbf{a} - 2\mathbf{b}$

(ii) $\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM} = 6\mathbf{b} + 2\mathbf{a} - 2\mathbf{b} = 2\mathbf{a} + 4\mathbf{b}$

(iii) $\overrightarrow{ML} = \overrightarrow{OL} - \overrightarrow{OM} = \dfrac{1}{2}\overrightarrow{OA} - \overrightarrow{OM}$

$= 3\mathbf{a} - (2\mathbf{a} + 4\mathbf{b}) = \mathbf{a} - 4\mathbf{b}$

(b) $\overrightarrow{OP} = \overrightarrow{OL} + \overrightarrow{LP} = \overrightarrow{OL} + 3\overrightarrow{LM} = \overrightarrow{OL} - 3\overrightarrow{ML}$

$= 3\mathbf{a} - 3(\mathbf{a} - 4\mathbf{b}) = 3\mathbf{a} - 3\mathbf{a} + 12\mathbf{b} = 12\mathbf{b}$

(c) Points O, B and P are collinear.
The distance from point B to point O is the same as that to point P.

(d) $\overrightarrow{ON} = \overrightarrow{OL} - \overrightarrow{NL} = \overrightarrow{OL} - \overrightarrow{BM}$

$= 3\mathbf{a} - (2\mathbf{a} - 2\mathbf{b}) = 3\mathbf{a} - 2\mathbf{a} + 2\mathbf{b} = \mathbf{a} + 2\mathbf{b}$