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ADDITIONAL MATHEMATICS 4038 PAPER 2 SOLUTIONS

1 (i)

$\sin(A-B)=\sin A\cos B-\cos A\sin B$

$\cos A\sin B=\sin A\cos B-\sin(A-B)=\frac{5}{8}-\frac{3}{8}=\frac{1}{4}$

(ii)

$\sin(A+B)=\sin A\cos B+\cos A\sin B=\frac{5}{8}+\frac{1}{4}=\frac{7}{8}$

(iii)

$\frac{\tan A}{\tan B}=\frac{\frac{\sin A}{\cos A}}{\frac{\sin B}{\cos B}}=\frac{\sin A\cos B}{\cos A\sin B}=\frac{\frac{5}{8}}{\frac{1}{4}}=\frac{5}{2}$

2 (i)

$$\frac{7}{2x^2-x-6}=\frac{7}{(2x+3)(x-2)}$$

$$=\frac{A}{2x+3}+\frac{B}{x-2}$$

$$7=A(x-2)+B(2x+3)$$

When $x=-\frac{3}{2},\ 7=-\frac{7}{2}A\Rightarrow A=-2$

When $x=2,\ 7=7B\Rightarrow B=1$

$$\therefore\frac{7}{(2x+3)(x-2)}=-\frac{2}{2x+3}+\frac{1}{x-2}$$

(ii)

$$\int_3^9\frac{7}{2x^2-x-6}\,dx=\int_3^9-\frac{2}{2x+3}+\frac{1}{x-2}\,dx$$

$$=\left[-2\frac{\ln|2x+3|}{2}+\ln|x-2|\right]_3^9$$

$$=\left[\ln\left|\frac{x-2}{2x+3}\right|\right]_3^9$$

$$=\ln\frac{(9)-2}{2(9)+3}-\ln\frac{(3)-2}{2(3)+3}$$

$$=\ln\frac{1}{3}-\ln\frac{1}{9}$$

$$=\ln 3$$

$$\approx 1.10$$

3 (i)

$8^x-2^{x+2}=15$

$2^{3x}-2^2(2^x)=15$

$u^3-4u-15=0$

(ii)

Let $f(u)=u^3-4u-15$

$f(3)=3^3-4(3)-15=0$

By Factor Theorem, $u-3$ is a factor of $f(u)$.

$f(u)=(u-3)(u^2+3u+5)$

$u=3$ or $u^2+3u+5=0$

Discriminant of $u^2+3u+5=0$ is $3^2-4(1)(5)=-1$

Since discriminant <0, $u^2+3u+5=0$ has no real solution.

$\therefore u=3$ is the only real solution of this equation.

(iii)

Comparing the coefficient of x^2,

$u=2^x=3$

$x=\log_2 3=1.58496\approx 1.58$

4 (i)

We first show that triangles ACP and BCQ are congruent. In triangles ACP and BCQ,

- $AC=BC$ (given)
- $PC=QC$ (given)
- $A\hat{C}P=B\hat{C}Q$ (common angle)

$\therefore$ Triangles ACP and BCQ are congruent.

$\Rightarrow C\hat{A}P=C\hat{B}Q\quad\cdots(1)$

Since triangle ABC is isosceles,

$C\hat{A}B=C\hat{B}A\quad\cdots(2)$

$X\hat{A}B=C\hat{A}B-C\hat{A}P$

$X\hat{B}A=C\hat{B}A-C\hat{B}Q$

By (1) and (2), $X\hat{A}B=X\hat{B}A$.

$\therefore AXB$ is an isosceles triangle.

(ii)

Since triangles ACP and BCQ are congruent,

$AP=BQ\quad\cdots(3)$

Since AXB is an isosceles triangle,

$AX=BX\quad\cdots(4)$

From the diagram,

$PX=AP-AX$

$QX=BQ-BX$

By (3) and (4), $PX=QX$.

5 (i)

$$\left(2-\frac{x}{4}\right)^n$$

$$=2^n+\binom{n}{1}2^{n-1}\left(-\frac{x}{4}\right)+\binom{n}{2}2^{n-2}\left(-\frac{x}{4}\right)^2+\ldots$$

$$=2^n-n(2^{n-1})\left(\frac{x}{4}\right)+\frac{n(n-1)}{2}(2^{n-2})\left(\frac{x^2}{16}\right)+\ldots$$

$$=2^n-2^{n-3}nx+2^{n-7}n(n-1)x^2+\ldots$$

(ii)

$$(1+x)\left(2-\frac{x}{4}\right)^n$$

$$=(1+x)(2^n-2^{n-3}nx+2^{n-7}n(n-1)x^2+\ldots)$$

$$\approx 2^n-2^{n-3}nx+2^{n-7}n(n-1)x^2+2^nx-2^{n-3}nx^2$$

$$=2^n-2^{n-3}nx+2^nx+2^{n-7}n(n-1)x^2-2^{n-3}nx^2$$

$$=2^n+(-2^{n-3}n+2^n)x+(2^{n-7}n(n-1)-2^{n-3}n)x^2$$

Comparing the coefficient of x,

$-2^{n-3}n+2^n=0$

$-\frac{1}{8}2^nn+2^n=0$

$2^n\left(-\frac{1}{8}n+1\right)=0$

$2^n=0$ (rej.) or $-\frac{1}{8}n+1=0$

$\therefore n=8$

(iii) Comparing the coefficient of the independent term,

$a = 2^n = 2^8 = 256$

Comparing the coefficient of x^2,

$b = 2^{n-7}n(n-1) - 2^{n-3}n$

$\quad = 2^{(8)-7}(8)((8)-1) - 2^{(8)-3}(8)$

$\quad = 2(8)(7) - 2^5(8)$

$\quad = -144$

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6 **(i)** $y = 1 + 2\cos x = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

Basic angle $x = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$

$\therefore x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$

Since $x_A < x_B$, the x-coordinate of A is $\frac{2\pi}{3}$.

The x-coordinate of B is $\frac{4\pi}{3}$.

(ii) Total area of the shaded regions

$= \int_0^{\frac{2\pi}{3}} 1 + 2\cos x \, dx - \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} 1 + 2\cos x \, dx$

$= [x + 2\sin x]_0^{\frac{2\pi}{3}} - [x + 2\sin x]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$

$= \left[\frac{2\pi}{3} + 2\sin\frac{2\pi}{3} - 0 - 2\sin 0\right]$

$-\left[\frac{4\pi}{3} + 2\sin\frac{4\pi}{3} - \frac{2\pi}{3} - 2\sin\frac{2\pi}{3}\right]$

$= \frac{2\pi}{3} + \sqrt{3} - \frac{4\pi}{3} - (-\sqrt{3}) + \frac{2\pi}{3} + \sqrt{3}$

$= 3\sqrt{3}$

≈ 5.20 square units

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7 **(i)** $y = |3x - 5| - 2$

When $x = 0$, $y = |-5| - 2 = 5 - 2 = 3$.

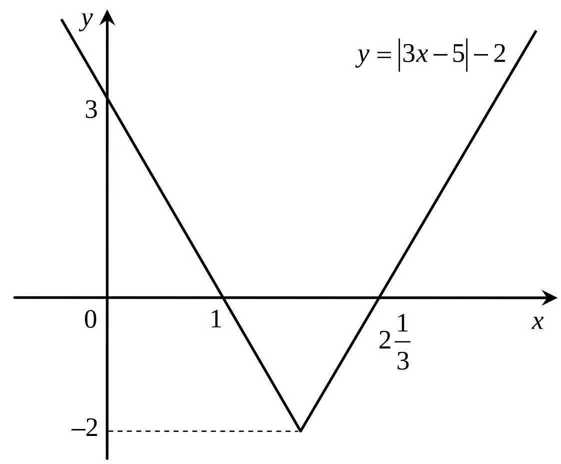
When $y = 0$, $3x - 5 = \pm 2$

$\Rightarrow 3x = 3 \quad$ or $\quad 3x = 7$

$\Rightarrow x = 1 \quad$ or $\quad x = 2\frac{1}{3}$

$\therefore$ The coordinates of the points are $(0,\ 3)$, $(1,\ 0)$ and $\left(2\frac{1}{3},\ 0\right)$.

(ii)



(iii) $x = |3x - 5| - 2$

$x + 2 = |3x - 5|$

$x + 2 = 3x - 5 \quad$ or $\quad -x - 2 = 3x - 5$

$2x = 7 \qquad 4x = 3$

$x = 3\frac{1}{2} \qquad x = \frac{3}{4}$

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8 **(i)** $s = 400(1 - e^{-\frac{t}{10}}) - 16t$

$v = \frac{ds}{dt} = 400\left(\frac{1}{10}e^{-\frac{t}{10}}\right) - 16 = 40e^{-\frac{t}{10}} - 16$

(ii) $a = \frac{dv}{dt} = 40e^{-\frac{t}{10}} - 16 = 40\left(\frac{1}{10}e^{-\frac{t}{10}}\right) = 4e^{-\frac{t}{10}}$

(iii) When $t = 0$, $v = 40e^0 - 16 = 24$ m s^{-1}

(iv) When $v = 0$, $40e^{-\frac{t}{10}} - 16 = 0$

$40e^{-\frac{t}{10}} = 16$

$e^{-\frac{t}{10}} = \frac{16}{40}$

$-\frac{t}{10} = \ln\frac{16}{40}$

$t = -10\ln\frac{16}{40}$

$= 9.1629$

≈ 9.163 seconds

(v) $s = 400(1 - e^{-\frac{9.163}{10}}) - 16(9.163)$

$= 93.393$

$\langle v \rangle = \frac{s}{t} = \frac{93.393}{9.163} = 10.2$ m s^{-1}

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9 **(i)** $(x-2)^2 + (y+1)^2 = 5^2$

$x^2 - 4x + 4 + y^2 + 2y + 1 = 25$

$x^2 + y^2 - 4x + 2y - 20 = 0$

$x^2 + y^2 - 2(2)x + 2(1)y - 20 = 0$

$\therefore g = -2,\ f = 1,\ c = -20$

(ii) The coordinates of A is $(-3, -1)$.

(iii) Gradient of AB = Gradient of $AO = \frac{1}{3}$

The equation of AB is $y = \frac{1}{3}x$.

(iv) Substitute $y = \frac{1}{3}x$ into $x^2 + y^2 - 4x + 2y - 20 = 0$.

$x^2 + \left(\frac{1}{3}x\right)^2 - 4x + 2\left(\frac{1}{3}x\right) - 20 = 0$

$x^2 + \frac{1}{9}x^2 - 4x + \frac{2}{3}x - 20 = 0$

$\frac{10}{9}x^2 - \frac{10}{3}x - 20 = 0$

$10x^2 - 30x - 180 = 0$

$x^2 - 3x - 18 = 0$

$(x+3)(x-6) = 0$

$x = -3$ (rej.) or $x = 6$

$y = \frac{1}{3}(6) = 2$

$\therefore$ The coordinates of B is $(6,\ 2)$.

10

$\frac{d^2y}{dx^2} = 6x - 6$

$\frac{dy}{dx} = \int 6x - 6 \, dx = 3x^2 - 6x + c_1$

When $x = 3$, $\frac{dy}{dx} = 12$, i.e.

$12 = 3(3)^2 - 6(3) + c_1 \Rightarrow c_1 = 3$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 6x + 3$

$y = \int 3x^2 - 6x + 3 \, dx = x^3 - 3x^2 + 3x + c_2$

The curve passes through the point (3, 10), i.e.

$10 = (3)^3 - 3(3)^2 + 3(3) + c_2 \Rightarrow c_2 = 1$

$\Rightarrow y = x^3 - 3x^2 + 3x + 1$

At the stationary point, $\frac{dy}{dx} = 3x^2 - 6x + 3 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

$y = x^3 - 3x^2 + 3x + 1$

$= (1)^3 - 3(1)^2 + 3(1) + 1$

$= 2$

The coordinates of the stationary point is (1, 2).

Since $\frac{d^2y}{dx^2} = 6(1) - 6 = 0$, we should test nearby points.

When $x = 0.9$, $\frac{dy}{dx} = 3(0.9)^2 - 6(0.9) + 3 = 0.03 > 0$

When $x = 1.1$, $\frac{dy}{dx} = 3(1.1)^2 - 6(1.1) + 3 = 0.03 > 0$

Since $\frac{dy}{dx} > 0$ when $x = 0.9$ and when $x = 1.1$, this is a point of inflexion.

11 **(i)** $C\hat{O}D = \theta$

$A\hat{O}B = A\hat{O}D - C\hat{O}D = 90° - \theta$

$O\hat{A}B = 90° - A\hat{O}B = 90° - (90° - \theta) = \theta$

$\cos\theta = \frac{AB}{17} \Rightarrow AB = 17\cos\theta$

$\sin\theta = \frac{OB}{17} \Rightarrow OB = 17\sin\theta$

$\cos\theta = \frac{OC}{31} \Rightarrow OC = 31\cos\theta$

$\sin\theta = \frac{CD}{31} \Rightarrow CD = 31\sin\theta$

$AB + BC + CD$

$= AB + (OC - OB) + CD$

$= 17\cos\theta + (31\cos\theta - 17\sin\theta) + 31\sin\theta$

$= 48\cos\theta + 14\sin\theta$

(ii) $AB + BC + CD = 49$

$48\cos\theta + 14\sin\theta = 49$

$= R\cos(\theta - \alpha)$

$\alpha = \tan^{-1}\frac{14}{48}$

$= 16.260°$

$R = \sqrt{48^2 + 14^2}$

$= 50$

$50\cos(\theta - 16.260°) = 49$

$\cos(\theta - 16.260°) = \frac{49}{50}$

Basic $(\theta - 16.260°) = \cos^{-1}\frac{49}{50}$

$= 11.478°$

$\theta - 16.260° = 11.478°,\ 360° - 11.478°$

$\theta \approx 27.7°,\ 364.8°$

$= 4.8°,\ 27.7°$

(iii) Maximum value of $AB + BC + CD$ is 50.

$\cos(\theta - 16.260°) = 1$

$\theta - 16.260° = 0$

$\theta = 16.260°$

$\approx 16.3°$