ADDITIONAL MATHEMATICS 4038 PAPER 1 SOLUTIONS

1

Let $f(x) = 2x^3 + ax^2 + bx + 3$.

Since $x-1$ is a factor of $f(x)$,

by the Factor Theorem, $f(1) = 0$, i.e.

$f(1) = 2(1)^3 + a(1)^2 + b(1) + 3 = 0$

$\Leftrightarrow a + b = -5 \quad \cdots(1)$

When divided by $x+2$, the remainder is 15.

By the Remainder Theorem, $f(-2) = 15$, i.e.

$f(-2) = 2(-2)^3 + a(-2)^2 + b(-2) + 3 = 15$

$\Leftrightarrow 4a - 2b = 28$

$\Leftrightarrow 2a - b = 14 \quad \cdots(2)$

Solving the simultaneous equations,

$(1)+(2): 3a = 9$

$\Rightarrow a = 3$

$b = -5 - 3 = -8$

2

$y = \dfrac{\ln x}{x}, \ x > 0$

$\dfrac{dy}{dx} = \dfrac{x\left(\dfrac{1}{x}\right) - \ln x(1)}{x^2}$

$= \dfrac{1-\ln x}{x^2}$

When y is an increasing function of x, $\dfrac{dy}{dx} > 0$, i.e.

$\dfrac{1-\ln x}{x^2} > 0$

$1 - \ln x > 0 \quad (\text{since } x^2 > 0)$

$\ln x < 1$

$x < e^1$

Hence, the set of values of x is

$\{x : 0 < x < e, \ x \in \mathbb{R}\}$

3

$x\sqrt{24} = x\sqrt{3} + \sqrt{6}$

$x\sqrt{24} - x\sqrt{3} = \sqrt{6}$

$x(\sqrt{24} - \sqrt{3}) = \sqrt{6}$

$x = \dfrac{\sqrt{6}}{\sqrt{24}-\sqrt{3}}$

$= \dfrac{\sqrt{6}}{\sqrt{24}-\sqrt{3}} \times \dfrac{\sqrt{24}+\sqrt{3}}{\sqrt{24}+\sqrt{3}}$

$= \dfrac{\sqrt{6\times 24} + \sqrt{6\times 3}}{24-3}$

$= \dfrac{\sqrt{3^2\times 16} + \sqrt{3^2\times 2}}{21}$

$= \dfrac{3(\sqrt{16}+\sqrt{2})}{21}$

$= \dfrac{4+\sqrt{2}}{7}$

$\therefore \ a = 4, \ b = 2$

4 **(i)**

$\lg(x+14) - \lg(x-2) = 2\lg 5$

$\lg\left(\dfrac{x+14}{x-2}\right) = \lg 5^2$

$\dfrac{x+14}{x-2} = 25$

$x + 14 = 25(x-2)$

$x + 14 = 25x - 50$

$24x = 64$

$x = 2\dfrac{2}{3}$

(ii)

$\log_2 y + \log_4 y = 6$

$\log_2 y + \dfrac{\log_2 y}{\log_2 4} = 6$

$\log_2 y + \dfrac{\log_2 y}{2} = 6$

$\dfrac{3\log_2 y}{2} = 6$

$\log_2 y = 4$

$y = 2^4$

$= 16$

5

$y = 1 - 3\tan x$

$\Rightarrow \dfrac{dy}{dx} = -3\sec^2 x$

When the curve crosses the y-axis, $x = 0$.

$y = 1 - 3\tan 0 = 1$

$\dfrac{dy}{dx} = -3\sec^2 0 = -3$

$\Rightarrow$ Gradient of normal $= \dfrac{1}{3}$

$\therefore$ Equation of normal is $y = \dfrac{1}{3}x + 1$

When $x = k$ and $y = 3$,

$3 = \dfrac{1}{3}k + 1$

$\therefore k = 6$

6 **(i)** $y = 2x^2 - 6x + c$

When $c = -20$,

$y = 2x^2 - 6x - 20$

$2x^2 - 6x - 20 \le 0$

$x^2 - 3x - 10 \le 0$

$(x+2)(x-5) \le 0$

When $x = 0,\ y = -20 \le 0$.

The set of values of x is $\{x : -2 \le x \le 5,\ x \in \mathbb{R}\}$.

(ii) $y = 2x^2 - 6x + c \quad \cdots(1)$

$y + 2x = 8$

$\Rightarrow y = 8 - 2x \quad \cdots(2)$

$(1) = (2) : 2x^2 - 6x + c = 8 - 2x$

$2x^2 - 4x + c - 8 = 0$

Discriminant $= (-4)^2 - 4(2)(c-8) = 0$

$16 - 8c + 64 = 0$

$8c = 80$

$\therefore c = 10$

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7 $x^2 + 2y^2 + 5x = 68 \quad \cdots(1)$

$2y + 3x = 9$

$\Rightarrow y = \dfrac{9-3x}{2} \quad \cdots(2)$

Sub (2) into (1) : $x^2 + 2\left(\dfrac{9-3x}{2}\right)^2 + 5x = 68$

$x^2 + 2\left(\dfrac{81 - 54x + 9x^2}{4}\right) + 5x = 68$

$2x^2 + 81 - 54x + 9x^2 + 10x = 136$

$11x^2 - 44x - 55 = 0$

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$x = -1 \quad \text{or} \quad x = 5$

The mid-point of the line is at $x = \dfrac{-1+5}{2} = 2$.

At this point, $y = \dfrac{9-3(2)}{2} = 1\dfrac{1}{2}$.

The coordinates of the mid-point is $\left(2,\ 1\dfrac{1}{2}\right)$.

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8 **(i)** LHS $= \cos 3x - \cos x$

$= -2\sin\tfrac{1}{2}(3x + x)\sin\tfrac{1}{2}(3x - x)$

$= -2\sin\tfrac{1}{2}(4x)\sin\tfrac{1}{2}(2x)$

$= -2\sin 2x \sin x$

$= -2(2\sin x\cos x)\sin x$

$= -4\sin^2 x\cos x$

= RHS

(ii) $\cos 3x + 2\cos x = 0$

$(\cos 3x - \cos x) + 3\cos x = 0$

$-4\sin^2 x\cos x + 3\cos x = 0$

$\cos x(-4\sin^2 x + 3) = 0$

$\cos x = 0 \quad \text{or} \quad -4\sin^2 x + 3 = 0$

$\Rightarrow x = \dfrac{\pi}{2} \qquad \Rightarrow \sin^2 x = \dfrac{3}{4}$

$\sin x = \pm\sqrt{\dfrac{3}{4}} = \pm\dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{3},\ \dfrac{2\pi}{3}$

$\therefore x = \dfrac{\pi}{3},\ \dfrac{\pi}{2} \text{ or } \dfrac{2\pi}{3}$

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9 **(i)** Since $-1 \le \sin\left(\dfrac{x}{3}\right) \le 1$,

Maximum value of $f(x)$ is $3(1) - 1 = 2$

Minimum value of $f(x)$ is $3(-1) - 1 = -4$

(ii) Amplitude of f is 3.

(iii) Period of f is $360° \div \dfrac{1}{3} = 1080°$

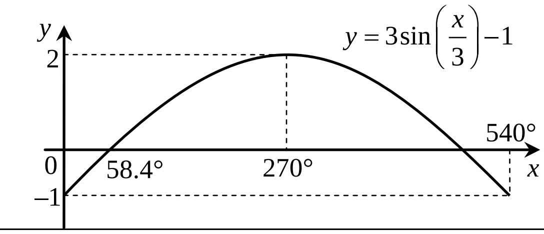
(iv) $f(x) = 3\sin\left(\dfrac{x}{3}\right) - 1 = 0$

$\sin\left(\dfrac{x}{3}\right) = \dfrac{1}{3}$

$\dfrac{x}{3} = \sin^{-1}\dfrac{1}{3} = 19.471°$

$x = 3 \times 19.471° \approx 58.4°$

(v)



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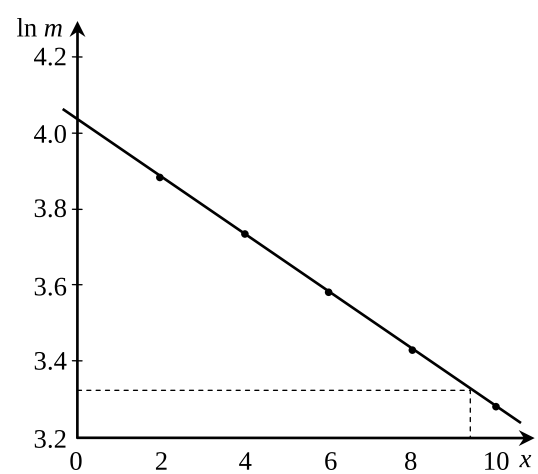
10 **(i)** Calculate the values of ln m.

t	2	4	6	8	10
ln m	3.88	3.73	3.58	3.42	3.28



(ii) $m = m_0 e^{-kt}$

$\ln m = \ln m_0 e^{-kt}$

$\ln m = \ln m_0 - kt \ln e$

$\ln m = \ln m_0 - kt$

$k = -\text{gradient} = -\left(\dfrac{3.31-3.91}{9.5-1.5}\right) = \dfrac{0.60}{8} = 0.075$

$\ln m_0 = 4.03 \Rightarrow m_0 = e^{4.03} = 56.26 \approx 56.3$

(iii) m_0 refers to the initial mass of the radioactive substance.

$\ln m = \ln \dfrac{m_0}{2} = \ln \dfrac{56.26}{2} = 3.337$

From the graph, $t = 9.2$ hours.

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11 (i) Gradient of $AD = \dfrac{6-(-2)}{0-2} = \dfrac{8}{-2} = -4$

Gradient of $AB = \dfrac{1}{4}$

Equation of AB is $y = \dfrac{1}{4}x + 6$

(ii) When $y = x$,

$x = \dfrac{1}{4}x + 6$

$\dfrac{3}{4}x = 6$

$x = 8$

$y = x = 8$

The coordinates of B is (8, 8).

(iii) $x_B - x_A = 8 - 0 = 8$

$y_B - y_A = 8 - 6 = 2$

$x_C - x_D = 2 \times 8 = 16$

$y_C - y_D = 2 \times 2 = 4$

$x_C = 16 + x_D = 16 + 2 = 18$

$y_C = 4 + y_D = 4 + (-2) = 2$

The coordinates of C is (18, 2).

(iv) Area of the trapezium $ABCD$

$= \dfrac{1}{2}\begin{vmatrix} 2 & 18 & 8 & 0 & 2 \\ -2 & 2 & 8 & 6 & -2 \end{vmatrix}$

$= \dfrac{1}{2}\left|(4+144+48+0)-(-36+16+0+12)\right|$

$= \dfrac{1}{2}\left|204\right|$

$= 102$ square units

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12 (i) $y = (2x-1)\sqrt{4x+1}$

$\dfrac{dy}{dx} = \dfrac{(2x-1)(4)}{2\sqrt{4x+1}} + 2\sqrt{4x+1}$

$= \dfrac{8x-4+4(4x+1)}{2\sqrt{4x+1}}$

$= \dfrac{8x-4+16x+4}{2\sqrt{4x+1}}$

$= \dfrac{24x}{2\sqrt{4x+1}}$

$= \dfrac{12x}{\sqrt{4x+1}}$

where $k = 12$.

(ii) When $x = 2$, $\dfrac{dy}{dt} = 2$, $\dfrac{dy}{dx} = \dfrac{12(2)}{\sqrt{4(2)+1}} = \dfrac{24}{3} = 8$

$\dfrac{dx}{dt} = \dfrac{dy}{dt} \div \dfrac{dy}{dx} = \dfrac{2}{8} = \dfrac{1}{4}$ units per second

(iii) $\displaystyle\int_0^2 \frac{3x}{\sqrt{4x+1}}\,dx$

$= \displaystyle\int_0^2 \frac{1}{4} \times \frac{12x}{\sqrt{4x+1}}\,dx$

$= \displaystyle\frac{1}{4}\int_0^2 \frac{12x}{\sqrt{4x+1}}\,dx$

$= \dfrac{1}{4}\left[(2x-1)\sqrt{4x+1}\right]_0^2$

$= \dfrac{1}{4}\left[(2(2)-1)\sqrt{4(2)+1} - (2(0)-1)\sqrt{4(0)+1}\right]$

$= 2\dfrac{1}{2}$
